Finite Elements: revision checklist

Colin Cotter

March 10, 2017

For the exam, you should be able to:

- Formulate one-dimensional boundary value problems as variational problems, with homogeneous and non-homogeneous Dirichlet and Neumann boundary conditions.
- Formulate 2D and 3D elliptic problems (Poisson, Helmholtz, variable coefficient versions) as variational problems, with homogeneous and non-homogeneous Dirichlet and Neumann boundary conditions.
- Construct Galerkin approximations for variational problems, and reformulate them as matrix-vector systems.
- Recall the general definition of Ciarlet's Finite Element.
- Define the nodal basis, and compute it in simple 1D examples.
- Recall definitions for 1D Lagrange elements on intervals, and 2D Lagrange elements on triangles.
- Recall the definition for a dual basis "determining" a finite element, and equivalence to the nodal basis being a basis for P.
- Recall the result about polynomials vanishing on a hyperplane, and use it to prove that dual bases determine finite elements for the case of triangular Lagrange elements, cubic Hermite elements, and other similar examples.
- Define the local interpolant for a finite element, and show that it is a linear operator, and a projection.
- Define the global interpolant for a triangulation with finite elements defined on each triangle, and show that the Lagrange elements are C^0. Understand the continuity of other examples of elements.
- Recall interpolation properties of the global interpolant for C^m functions.
- Recall the definition of the L^2 norm, understand the definition of L^2 spaces as sets of equivalence classes.
- Recall the definition of the H^k norms and semi-norms.
- Recall the Schwarz inequality.
- Define C_0^∞, L^1_{loc}, and use them to define a weak derivative.
- Use the definition to compute weak derivatives of continuous functions, and finite element functions.
- Recall the Sobolev norms, semi-norms and spaces, and recall that $W_p^k(\Omega)$ is a Hilbert space.
- Recall Sobolev's inequality, and the fact that C^∞ is dense in H^k.
- Define inner-product spaces, and recall that L^2 and $W_2^k = H^k$ are inner product spaces.
- Recall Schwarz's inequality for inner product spaces.
- Understand how to use an inner-product space as a normed space.
- Recall the definition of a Hilbert space, and recall that L^2 and H^k are Hilbert spaces.
- Define the dual of a Hilbert space, and recall the Riesz Representation theorem.

- Define non-symmetric variational problems, and recall the Lax-Milgram theorem for non-symmetric problems and their Ritz-Galerkin approximations.
- Recall Cea's Lemma, and the proof.
- Prove the trace inequality for finite element functions, and consequently for H^k functions.
- Proe the mean inequality for finite element functions.
- Recall approximation theory results for averaged Taylor polynomials and use them to obtain convergence estimates for variational problems via Cea's Lemma.
- Deduce the mean inequality for H^k functions from the approximation theory results.
- Recall results on integration by parts for H^1 spaces.
- Formulate Poisson's equation as a variational problem, with Dirichlet and Neumann's boundary conditions.
- Use the mean inequality to derive a coercivity result for Poisson's equation with Neuman boundary conditions.
- Use the mean inequality and trace inequality to derive a coercivity result for Poisson's equation with Dirichlet boundary conditions.
- Show that the Galerkin approximation for Poisson's equation converges.